

On the observability of Bell's inequality violation in the optical Stern-Gerlach model

M. Tumminello [†], A. Vaglica [‡], G. Vetri [‡]

[†]*Istituto Nazionale di Fisica della Materia and Dipartimento di Fisica e Tecnologie Relative,
Università degli Studi di Palermo, Viale delle Scienze, Palermo, I-90128, Italy*

[‡]*Istituto Nazionale di Fisica della Materia and Dipartimento di Scienze Fisiche ed Astronomiche,
Università degli Studi di Palermo, via Archirafi 36, 90123 Palermo, Italy*

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Using the optical Stern-Gerlach model, we have recently shown that the non-local correlations between the internal variables of two atoms that successively interact with the field of an ideal cavity in proximity of a nodal region are affected by the atomic translational dynamics. As a consequence, there can be some difficulties in observing violation of the Bell's inequality for the atomic internal variables. These difficulties persist even if the atoms travel an antinodal *region*, except when the spatial wave packets are exactly centered in an antinodal *point*.

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Peculiar concepts of quantum mechanics (QM), such as the Bohr's principle of complementarity [1, 2, 3, 4], have their origin in the vectorial nature of the state space, which involves a superposition principle. Complementarity (or duality) [5] establishes a sort of "orthogonality" between the which-way information and the possibility of observing interference pattern. In other words, these two behaviors are mutually exclusive. The visibility V of the interference pattern and the distinguishability D of the quantum paths can in some extent coexist, and as shown by Englert in its quantitative analysis of complementarity [5, 6], they satisfy the inequality $D^2 + V^2 \leq 1$. According to this analysis and in the ambit of the optical Stern-Gerlach (SG) model, we have recently shown [7] that the visibility of the Rabi oscillations and the distinguishability of the two atomic translational paths satisfy the equality relation $D^2 + V^2 = 1$ when pure initial states are considered.

When applied to a composite system, the superposition principle leads to quantum correlations (entanglement), which may hide the individuality of the subsystems. Differently from the classical case, and in idealized configurations, two quantum systems that have interacted for a time, generally do not recover their individuality, even if the subsystems become spatially separated. This inseparability, which has been at the origin of the famous debate between Einstein [8] and Bohr [9] on the completeness of QM, implies a non local character of the correlations (EPR correlation [10]) between the two subsystems. This non-locality can be individuated by the violation of some Bell's inequality [11]. It is to note that, differently from the pure case, a mixed state may be EPR correlated and, at the same time, it may satisfy the Bell's inequality [10].

Recently, it has been paid attention to teleportation, non-local correlations, separability and related issues for massive particles [12, 13, 14, 15, 16, 17, 18, 19, 20]. As suggested by Phoenix and Barnett [15] (see also [12] and [16]), a simple model which can realize an EPR state for massive particles consists of two atoms which interact

successively with the field of an optical cavity, in the ambit of the standard Jaynes-Cummings (JC) model. The entanglement developed during the interaction between the first atom and the field, may induce quantum correlations between the two atoms as the second one interacts with the field of the same cavity. An experimental effort to observe a Bell's inequality violation for this system has been done by Haroche and co-workers [12], which ascribe to several experimental imperfections the reduction of purity of the entanglement that prevents the Bell's inequality violation.

In a recent paper [20] we have suggested that a careful analysis of the interatomic correlations may require the quantization of the translational dynamics of the two atoms along the cavity axis. In that paper we have considered two-level atoms entering the cavity in a nodal region, where the field gradient is different from zero. The entanglement between the internal and external atomic variables affects the non local features of the interatomic correlations making more difficult the observation of Bell's inequality violation with respect to the JC model. However, in most cases (as in ref.: [12]) the experiments are performed in such a way that the atoms interact with the cavity field in an antinodal region. Consequently, it seems suitable to extend our previous analysis to this case. Our present analysis confirms qualitatively the results of the previous one, except when the spatial atomic wave packets are exactly centered in an antinodal *point*.

In our model two two-level atoms interact successively with a single mode of the e.m. field of an ideal cavity. The first atom, say A_1 , enters the cavity at time $t = 0$ and interacts with the field for a time t_1 . It moves prevalently along the z -direction, orthogonal to the x -cavity axis and we assume that the velocity along this direction is large enough to treat classically this component of the motion. The second atom, say A_2 , enters the cavity at time $t_2 > t_1$, interacts with the e.m. field as modified by the first atom and leaves the cavity at time t_3 . Finally, both the

atoms evolve freely for $t > t_3$. The atoms enter the cavity in proximity of an antinodal region of the resonant k -mode, and the width of their wave packets is sufficiently

small with respect to the wavelength of this mode. The Hamiltonian of the system at all times can consequently be written as

$$\hat{H} = \frac{\hat{p}_1^2}{2m} + \hbar\omega(\hat{a}^\dagger\hat{a} + \hat{S}_{z,1} + \frac{1}{2}) + (\frac{\hat{p}_2^2}{2m} + \hbar\omega\hat{S}_{z,2})\theta_t(t_2) + \hbar\varepsilon(\frac{k^2\hat{x}_1^2}{2} - 1)\mu_t(0, t_1)\hat{u}_1 + \hbar\varepsilon(\frac{k^2\hat{x}_2^2}{2} - 1)\mu_t(t_2, t_3)\hat{u}_2, \quad (1)$$

where \hat{x}_i is the position of atom A_i with respect to the antinodal point and \hat{p}_i its conjugate momentum. The atom-field interaction is described by $\hat{u}_i = \hat{a}^\dagger\hat{S}_{-,i} + \hat{a}\hat{S}_{+,i}$ where \hat{a} and \hat{a}^\dagger are the usual annihilation and creation field-operators, while $\hat{S}_{\pm,i}$ are the 1/2 spin operators. The atoms have same mass m and same atom-field coupling constant ε . The linear combination of step-functions $\mu_t(x, y) = \theta_t(x) - \theta_t(y)$, with different points (x and y) of discontinuity, distinguishes the different time ranges concerning the successive interactions.

As in Ref. [15] where the standard JC model is adopted and as in our previous paper [20], we consider the simple case of only one atom-field system excitation. In particular, we start considering both the atoms initially in the ground state and just one photon in the cavity, so at time $t = 0$ the state is $|\psi(0)\rangle = |g_1\rangle|1\rangle|\varphi_1(0)\rangle$, where $|\varphi_1(0)\rangle$ is a translational state of the atom A_1 .

Using the evolution operator related to eq.(1), the state of the system for $t \leq t_2$ is (except an irrelevant global phase factor)

$$|\psi(t)\rangle = \exp[-\frac{i}{\hbar}\frac{\hat{p}_1^2}{2m}(t - t_1)] \cdot [|S_1^-(t_1)\rangle|e_1\rangle|0\rangle + |S_1^+(t_1)\rangle|g_1\rangle|1\rangle], \quad (2)$$

where $|e_i\rangle$ indicates the excited state of A_i and

$$|S_1^\pm(t_1)\rangle = \frac{1}{2}[e^{i\varepsilon t_1}|\phi_1^+(t_1)\rangle \pm e^{-i\varepsilon t_1}|\phi_1^-(t_1)\rangle] \quad (3)$$

$$|\phi_1^\pm(t_1)\rangle = \exp\{-\frac{i}{\hbar}[\frac{\hat{p}_1^2}{2m} \pm \hbar\varepsilon k^2\frac{\hat{x}_1^2}{2}]t_1\}|\varphi_1(0)\rangle. \quad (4)$$

At time $t = t_2$ the second atom, in its ground internal state, enters the cavity and starts to interact with the field modified by the interaction with the first atom. Let $|\varphi_2(t_2)\rangle$ be the translational state of atom A_2 at the beginning of its interaction with the cavity field. The state of the entire system at this time is $|\Psi(t_2)\rangle = |\psi(t_2)\rangle|g_2\rangle|\varphi_2(t_2)\rangle$. Applying the same procedure as above, we derive the state at time $t > t_3$, when both the two atoms have left the cavity and evolve freely

$$|\Psi(t)\rangle = |S_1^+(t)\rangle|S_2^+(t)\rangle|g_1\rangle|g_2\rangle|1\rangle + \{ |S_1^-(t)\rangle|\varphi_2(t)\rangle|e_1\rangle|g_2\rangle + |S_1^+(t)\rangle|S_2^-(t)\rangle|g_1\rangle|e_2\rangle \} |0\rangle \quad (5)$$

where

$$|S_i^\pm(t)\rangle = \exp[-\frac{i}{\hbar}\frac{\hat{p}_i^2}{2m}(t - t_3)]|S_i^\pm(t_3)\rangle \quad (6)$$

$$|S_i^\pm(t_3)\rangle = \frac{1}{2}[e^{i\varepsilon T_i}|\phi_i^+(t_3)\rangle \pm e^{-i\varepsilon T_i}|\phi_i^-(t_3)\rangle] \quad (7)$$

$$|\phi_1^\pm(t_3)\rangle = \exp[-\frac{i}{\hbar}\frac{\hat{p}_1^2}{2m}(t_3 - t_1)]|\phi_1^\pm(t_1)\rangle \quad (8)$$

$$|\phi_2^\pm(t_3)\rangle = \exp\{-\frac{i}{\hbar}[\frac{\hat{p}_2^2}{2m} \pm \hbar\varepsilon k^2\frac{\hat{x}_2^2}{2}]T_2\}|\varphi_2(t_2)\rangle \quad (9)$$

$$|\varphi_2(t_3)\rangle = \exp[-\frac{i}{\hbar}\frac{\hat{p}_2^2}{2m}T_2]|\varphi_2(t_2)\rangle, \quad (10)$$

and we have introduced the interaction time $T_1 = t_1$ and $T_2 = t_3 - t_2$ for atoms A_1 and A_2 , respectively. Tracing on the field and atomic translational variables, the following reduced density operator is obtained

$$\rho = Tr_{f,s_1,s_2}(|\Psi(t_3)\rangle\langle\Psi(t_3)|) = \frac{1}{4}(1 + c_R^{(1)})(1 + c_R^{(2)})|g_1\rangle|g_2\rangle\langle g_1|\langle g_2| + \frac{1}{2}(1 - c_R^{(1)})|e_1\rangle|g_2\rangle\langle e_1|\langle g_2| + \frac{1}{4}(1 + c_R^{(1)})(1 - c_R^{(2)})|g_1\rangle|e_2\rangle\langle g_1|\langle e_2| + \frac{i}{4}c_I^{(1)}[(c_- - c_+)|e_1\rangle|g_2\rangle\langle g_1|\langle e_2| - h.c.], \quad (11)$$

where we have put

$$\begin{aligned} e^{-2i\varepsilon T_1}\langle\phi_1^+(t_1)|\phi_1^-(t_1)\rangle &= c_R^{(1)} + ic_I^{(1)} \\ e^{-2i\varepsilon T_2}\langle\phi_2^+(t_3)|\phi_2^-(t_3)\rangle &= c_R^{(2)} + ic_I^{(2)} \\ e^{\mp i\varepsilon T_1}\langle\phi_2^\pm(t_3)|\varphi_2(t_3)\rangle &= c_\pm. \end{aligned} \quad (12)$$

As it is easy to see, eq. (11) is formally very similar to the corresponding equation of Ref.[15]. The difference between eq. (11) and the corresponding in Ref.[15] is the fact that the coefficients (12) are now affected by

the translational dynamics. As in Ref.[20], the scalar products which appear in this equation are generally subjected to a non dissipative decay and this behavior may affect the non local character of the correlations between the internal atomic variables.

An evaluation of the quantities (12) is not a trivial operation because the evolution operator in eq.s (4) and (9) describes a harmonic-like evolution (sign $-$) or a squeezing-like evolution in the other case (sign $+$) [23]. In fact one can write

$$e^{-\frac{i}{\hbar}\left(\frac{\hat{p}_i^2}{2m} - \frac{\hbar\epsilon k^2}{2}\hat{x}_i^2\right)T_i} = e^{i\frac{\omega_0}{2}T_i(\hat{b}_i^{\dagger 2} + \hat{b}_i^2)}, \quad (13)$$

where T_i is the interaction time of atom A_i ,

$$\hat{b}_j = \frac{1}{\sqrt{2}}\left(\sqrt{\frac{m\omega_0}{\hbar}}\hat{x}_j + i\frac{1}{\sqrt{m\omega_0\hbar}}\hat{p}_j\right) \quad (14)$$

are boson operators and $\omega_0^2 = \frac{\hbar k^2}{m}\epsilon$. To calculate the scalar products (12), it is convenient to put the squeezing operators (13) in factored forms [21], for instance

$$e^{i\frac{\alpha}{2}(\hat{b}_i^{\dagger 2} + \hat{b}_i^2)} = \exp\{-\ln[\cosh(\alpha)](\hat{b}_i^{\dagger} \hat{b}_i + \frac{1}{2})\} \cdot \exp\{\frac{i}{2}\tanh(\alpha)e^{2\ln[\cosh(\alpha)]}\hat{b}_i^{\dagger 2}\} \cdot \exp\{\frac{i}{2}\tanh(\alpha)\hat{b}_i^2\}, \quad (15)$$

and similar expressions. Moreover, for the sake of mathematical simplicity, we assume that the initial trans-

lational states for both the atoms are given by coherent states of the boson-like operators b_j , with the same width. In other words, we suppose that at the beginning of the interaction with the cavity field the translational states of both the atoms are coherent states with respect to the bosons operators \hat{b}_j : $|\varphi_j(initial)\rangle = |\alpha_j\rangle$, with $\hat{b}_j|\alpha_j\rangle = \alpha_j|\alpha_j\rangle$, and

$$|\alpha_j\rangle = \exp\left[\frac{i}{\hbar}\left(p_0^{(j)}\hat{x}_j - x_0^{(j)}\hat{p}_j\right)\right]|0_j\rangle \quad (16)$$

$$\alpha_j = x_0^{(j)}\sqrt{\frac{m\omega_0}{2\hbar}} + ip_0^{(j)}\frac{1}{\sqrt{2m\hbar\omega_0}} \equiv a_j + ib_j \quad (17)$$

where

$$\langle x_j|0_j\rangle = \left(\frac{1}{\Delta x_0\sqrt{2\pi}}\right)^{\frac{1}{2}} \exp\left[-\frac{x_j^2}{4\Delta x_0^2}\right] \quad (18)$$

is the wave function of the ground state of the \hat{b}_j corresponding harmonic “oscillator” and $\Delta x_0^2 = \frac{\hbar}{2m\omega_0}$ is the same for both the atoms. This choice is not too restrictive because the only restriction introduced with respect to a minimum uncertainty gaussian packet with arbitrary initial momentum p_0 and position x_0 is its wideness. Furthermore, it is to notice that a general packet can always be expressed as a superposition of coherent states. Using eq.s (15) and (16), the scalar products which appear in eq. (12) assume the following form,

$$\langle\phi_1^+(T_1)|\phi_1^-(T_1)\rangle = \langle\alpha_1|e^{i\omega_0 T_1 - \ln[\cosh(\omega_0 T_1)]}\{\hat{b}_1^{\dagger} \hat{b}_1 + \frac{1}{2}\}e^{\frac{i}{4}\sinh(2\omega_0 T_1)\hat{b}_1^{\dagger 2}}e^{\frac{i}{2}\tanh(\omega_0 T_1)\hat{b}_1^2}|\alpha_1\rangle \quad (19)$$

$$\langle\phi_2^+(T_2)|\varphi_2(T_2)\rangle = \langle\alpha_2|e^{i\omega_0 T_2}\{\hat{b}_2^{\dagger} \hat{b}_2 + \frac{1}{2}\}e^{-i\frac{\hat{p}_2^2}{2m\hbar}T_2}|\alpha_2\rangle \quad (20)$$

$$\langle\phi_2^-(T_2)|\varphi_2(T_2)\rangle = \langle\alpha_2|e^{-\frac{i}{2}\tanh(\omega_0 T_2)\hat{b}_2^{\dagger 2}}e^{-\frac{i}{4}\sinh(2\omega_0 T_2)\hat{b}_2^2}e^{-\ln[\cosh(\omega_0 T_2)]}\{\hat{b}_2^{\dagger} \hat{b}_2 + \frac{1}{2}\}e^{-i\frac{\hat{p}_2^2}{2m\hbar}T_2}|\alpha_2\rangle \quad (21)$$

A straightforward calculation leads now to the evaluation of these scalar products, where the expansion of the state $\exp[-i\frac{\hat{p}_2^2}{2m\hbar}t]|\alpha_2\rangle$ in terms of coherent states corresponding to the second atom boson-like operators is

required.

For $x_0 \neq 0$ and/or $p_0 \neq 0$ these terms are characterized by a non dissipative damping. For example, the scalar product (19) for $t \leq T_1$ behaves as

$$\begin{aligned} \langle\phi_1^+(t)|\phi_1^-(t)\rangle &= e^{i\frac{\omega_0}{2}t}e^{-|\alpha_1|^2\frac{\sinh(\omega_0 t)}{\cosh(\omega_0 t)}}\exp\left\{\frac{i}{2}\tanh(\omega_0 t)[(a_1^2 - b_1^2)(1 + \cos(2\omega_0 t)) + 2a_1 b_1 \sin(2\omega_0 t)]\right\} \cdot \\ &\cdot \frac{1}{\sqrt{\cosh(\omega_0 t)}}e^{-|\alpha_1|^2(1 - \frac{\cosh(\omega_0 t)}{\cosh(\omega_0 t)})}\exp\{-\tanh(\omega_0 t)[a_1 b_1(1 - \cos(2\omega_0 t)) + \frac{1}{2}(a_1^2 - b_1^2)\sin(2\omega_0 t)]\} \end{aligned} \quad (22)$$

$$\propto \left[1 - \frac{(\omega_0 t)^2}{2}\right] \cdot \exp\{-2a_1^2(\omega_0 t)^2\} \quad (\omega_0 t < 1) \quad (23)$$

The damping factor shown by this last approximated ex-

pression, which is at the origin of the non dissipative

damping of the Rabi oscillations [7, 22, 23], is due to the increasing distance in the phase space [24] of the two deflected components of the translational wave packet [25]. Similar behaviors hold for the other coefficients of eq.(12). The condition ($\omega_0 t < 1$) is not much restrictive for the parameters used in this paper, and at the same time, it is in agreement with the quadratic approximation of the cavity mode function. After a few periods of Rabi oscillations, the damping factors involved in the scalar products (12) when $x_0 \neq 0$, determine a decoherence of the system described by the density matrix (11), i.e. the last term in eq. (11), representing the non-diagonal terms, goes to zero. The system tends to become separable [10]. It is impossible to observe such a behavior in the JC model context.

It is to notice that when both the atoms interact with the field exactly in coincidence of the antinode (i.e. $x_0 = 0$), the scalar products (19-21) produce just a slow damping of the correlation functions and the density matrix remain essentially non-separable.

Because, as said above, non-separability doesn't imply a violation of Bell's inequality, guarantee of non-locality, it is also useful to investigate the nature of the interatomic correlations in terms of the Bell's inequality. To this end we consider the Horodecki family formulation [26]: *A density matrix ρ describing a system composed by two spin 1/2 subsystems violates some Bell's inequality in the CHSH formulation [27] if and only if the relation $M(\rho) > 1$ is satisfied.* The quantity $M(\rho)$ is defined as follows. Consider the 3×3 matrix T_ρ with coefficients $t_{n,m} = \text{tr}(\rho \sigma_n \otimes \sigma_m)$, where σ_n are the standard Pauli matrices. Diagonalizing the symmetric matrix $U_\rho = T_\rho^T \cdot T_\rho$ (T_ρ^T is the transpose of T_ρ), and denoting the three eigenvalues of U_ρ by λ_1, λ_2 and λ_3 , then $M(\rho) = \max\{\lambda_1 + \lambda_2, \lambda_1 + \lambda_3, \lambda_2 + \lambda_3\}$. In our case $\lambda_2 = \lambda_3$ and then $M(\rho) = \max\{\lambda_1 + \lambda_2, 2\lambda_2\}$. Fig. 1 compares the behaviors of $\lambda_1 + \lambda_2$ (continuous line) and $2\lambda_2$ (dashed line) as a function of the interaction time for the two models. For simplicity, in both the figures (i) and (ii) we have assumed $T_1 = T_2 = T$. The response of the Bell's inequality test outlines the great difference between the interatomic correlations predicted by the two models when $x_0 \neq 0$. When $x_0 = 0$ the JC and SG models conduce to an almost indistinguishable behavior of the system with respect to non-locality (see Fig. 1 (i)). This is due to the fact that the eq. (22) reduces to $1/\sqrt{\cosh(\omega_0 t)}$ when $x_0 = p_0 = 0$ and this term results slowly decaying for our values of parameters also in comparison with the decay in eq. (23).

It is possible, furthermore, to extend the discussion to another simple case in which the single excitation belongs initially to the atom A_1 . For this initial state, the quantity $M(\rho)$ reduces simply to the dashed line of figures (i) and (ii).

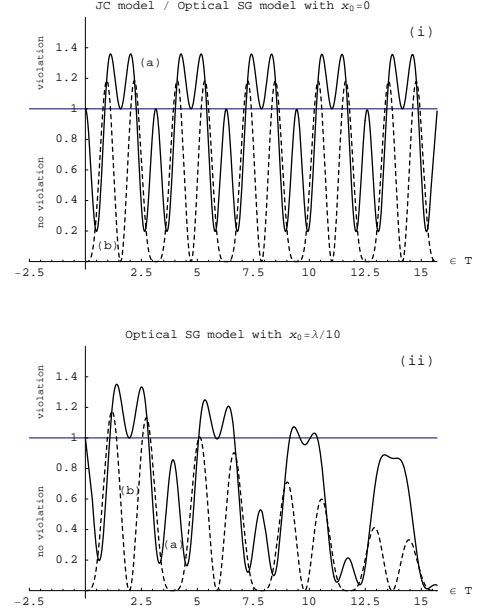


FIG. 1: Graphical solution of Bell's inequality in terms of the $M(\rho) = \max\{(a), (b)\}$ function, for two two-level atoms interacting in succession with the field of the same cavity. Figure (i) shows the periodicity of $\lambda_1 + \lambda_2$ (continuous line) and $2\lambda_2$ (dashed line) when the JC model is adopted Ref.[15] or the SG model is considered with $x_0 = 0$ for both the atoms. Fig. (ii) illustrates the non dissipative damping of the correlation between the two atoms due to the entanglement of the field and the internal atomic variables with the translational atomic degrees of freedom when $x_0 \neq 0$. For little interaction times a magnification of the non locality of the entanglement is observed. Concerning the translational dynamics, in this graph we suppose for both the atoms an initial wave packet of minimum uncertainty, with zero mean value of \hat{p}_1 and \hat{p}_2 , centered in $x_1 = x_2 = \lambda/10$ and with a width imposed by the condition of dealing with coherent initial states. $\lambda = 2\pi/k$ is the wavelength of the resonant k-mode of the undamped cavity. The values of the other parameters are $m = 10^{-26}$ kg, $\epsilon = 10^8 \text{ sec}^{-1}$ and $\lambda = 10^{-5}$ meters.

In conclusion, the internal variables of two atoms that successively cross an optical cavity may result strongly entangled through the interaction with the field of the same cavity. This entanglement may lead to Bell's inequality violation. As it is known, transfer of information from the system of interest to other degrees of freedom (to a bath, in the extreme case) produces a degradation of quantum correlations. For the system here considered (the optical SG model) the correlation with the atomic translational degrees of freedom can actually be avoided by letting the atoms cross as accurately as possible the cavity in the region with a zero gradient of the mode function.

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